A Study on the Labeling of Certain Graphs using Prime Correspondence

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Abstract: Let $G = (p, q)$ graph. A bijection $f: V(G) \longrightarrow \{1,2,3...p\}$ induces an edge labeling $f^*: E(G) \longrightarrow \{0,1\}$ such that for any edge $uv \in E(G)$ $f^*(uv) = 1,$ if $gcd\ (S, D) = 1,$ $f^*(uv) = 0,$ otherwise. where $S = f(u) + f(v)$ and $D = |f(u) - f(v)|$
we say that f is SD- prime labeling if $f^*(uv) = 1$ for all $uv \in E(G)$. G is SD prime if it admits SD prime labeling.The labeling f is called SD prime cordial labeling if it satisfies $|e_{f^*(1)} - e_{f^*(0)}| \le 1$, where $e_{f^*(1)}, e_{f^*(0)}$ is number of edges labeled by 1 and 0 respectively. G is SD prime cordial if it admits SD prime cordial labeling. In this paper we proved that $B_n, O(TL_n), D_2(P_n)$ and $D_2(C_n)$ if $n \ge 3, n\ \ odd$, admits SD Prime Cordial. A graph that admits SD prime cordial labeling is called SD prime cordial graphs.

Keywords: Path and Cycle; Brush graph; Shadow graph; Triangular graph; SD prime labeling; SD prime cordial labeling

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1 Introduction

Consider a graph $G = (V(G), E(G))$ be a simple finite and undirected with order $|V| = p$ and size $|E| = q$, the notation can be found in [1]. In [2] refer for detailed survey of graph labeling. In [3] and [4] the authors introduced the concept of SD- Prime cordial labeling and they proved for graphs like fan, star, wheel, double star, path, ladder, double fan. In [5] and [6] the authors proved by duplication of each vertex of path and cycle by an edge admits SD-Prime Cordial labeling . In this paper we prove that $B_n, O(TL_n), D_2(P_n)$ and $D_2(C_n)$ if $n \ge 3, n\ \ odd$ are SD-Prime Cordial graphs.

II. Preliminaries

Definition 1.1 *A bijection $f: V(G) \longrightarrow \{1,2, \cdots |V(G)|\}$ induces an edge labeling $f^*: E(G) \longrightarrow \{0,1\}$ such that for any edge uv in G. $f^*(uv) = 1$ if $\gcd(S, D) = 1$ and $f^*(uv) = 0$ otherwise we say f is SD-Prime labeling in $f^*(uv) = 1$ for all $uv \in E(G)$. Moreover G is SD-Prime if it admits SD-Prime labeling.*

Definition 1.2 *A bijection $f: V(G) \longrightarrow \{1,2,3, \cdots |V(G)|\}$ induces an edge labeling $f^*: E(G) \longrightarrow 0,1$ such that for any edge uv in G, $f^*(uv) = 1$ if gcd $(S, D) = 1$ and $f^*(uv) = 0$ otherwise. The labeling f is called SD-Prime Cordial labeling if $|e_{f^*(1)} - e_{f^*(0)}| \le 1$. We say that G is SD-Prime cordial if it SD-Prime cordial labeling.*

Definition 1.3 Brush Graph

The Brush graph $B_n, (n \ge 2)$ can be constructed by path graph $P_n, (n \ge 2)$ by joining the star graph $k_{1,1}$ at each vertex of the path . (ie) $B_n = P_n + nK_{1,1}$.

Definition 1.4 Open Triangular Ladder Graph

An Open Triangular Ladder $O(TL_n), n \ge 2$ is obtained from an open ladder $O(L_n)$ by adding the $\{u_i v_{i+1}, 1 \le i \le n-1\}$.

Definition 1.5 Shadow Graph

Let G be a connected graph. A graph constructed by taking two copies of G say G_1 and G_2 and joining each vertex u in G_1 to the neighbours of the corresponding vertex u in G_1 there exits v in G_2 such that $N(u) = N(v)$. The resulting graph is known as shadow graph and it is denoted by $D_2(G)$.

Definition 1.6 Path

All the vertices in a walk are distinct is called a path and a path of length k is denoted by P_{k+1}.

Definition 1.7 Cycle

A closed path is called a cycle ,and path of length

k is denoted by C_k.

II Main Results

Theorem 2.1. The Brush graph $B_n, n \geq 2$ is SD-Prime cordial .

Proof: Let $G = B_n$ the brush graph

Let

$V(G) = \{v_i / 1 \leq i \leq n\} U \{u_i / 1 \leq i \leq n\}$

Let

$E(G) = \{u_i u_{i+1} / 1 \leq i \leq n-1\} U \{u_i v_i / 1 \leq i \leq n\}$

$|V(B_n)| = 2n, |E(B_n)| = 2n - 1$

Define a vertex labeling ,

$f: V(B_n) \longrightarrow \{1,2,3, \cdots, 2n\}$

$f(u_i) = 2i - 1, 1 \leq i \leq n$

$f(v_i) = 2i, 1 \leq i \leq n$

The induced edge labeling is $f^*: E(G) \longrightarrow \{0,1\}$ defined by

$f^*(uv) = 1$, if $gcd(S, D) = 1$,

$f^*(uv) = 0, otherwise$

The edge sets are

$E_1 = \{u_i u_{i+1}, \quad 1 \leq i \leq n-1\}$

$E_2 = \{u_i v_i, \quad 1 \leq i \leq n\}$

In E_1

$$GCD \; [S, D] = GCD \; [f(u_i) + f(u_{i+1}), |f(u_i) - f(u_{i+1})|]$$

$$= GCD \; [2i - 1 + 2(i+1) - 1, |2i - 1 - (2(i+1) - 1)|]$$

$$= GCD \; [2i - 1 + 2i + 2 - 1, |2i - 1 - (2i + 2 - 1)|]$$

$$= GCD \; [4i, 2], \quad 1 \leq i \leq n-1$$

$$\neq 1$$

$$f^*(u_i u_{i+1}) = 0, \quad 1 \leq i \leq n-1$$

In E_2

$$GCD \; [S, D] = GCD \; [f(u_i) + f(v_i), |f(u_i) + f(v_i)|]$$

$$= GCD \; [2i - 1 + 2i, |2i - 1 - 2i|]$$

$$= GCD \; [4i - 1, 1], \quad 1 \leq i \leq n$$

$$= 1$$

$$f^*(u_i v_i) = 1, \quad 1 \leq i \leq n$$

Thus $\left| e_{f^*(0)} - e_{f^*(1)} \right| \leq 1$

Hence B_n admits SD-rime cordial labeling.

$\therefore B_n$ is prime cordial.

Illustration 2.2

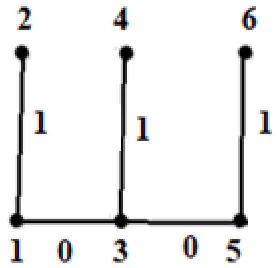

Figure 1 B_3

Theorem 2.3. The graph $O(TL_n)$ $n \geq 2$ admits SD-Prime cordial labeling.

proof: Let $G = O(TL_n)$ the open triangular ladder graph.

Let $V(G) = \{u_i / 1 \leq i \leq 2n\}$

Let

$E(G) = \{u_{2i-1} u_{2i+1} / 1 \leq i \leq n-1\} \cup \{u_{2i} u_{2i+2} / 1 \leq i \leq n-1\} \cup$

$\{u_{2i} u_{2i+1} / 1 \leq i \leq n-1\} \cup \{u_{2i+1} u_{2i+2} / 1 \leq i \leq n-1\}$

$|V(O(TL_n))| = 2n$

$|E(O(TL_n))| = 4n - 4$

Define a labeling,

$f: V(O(TL_n)) \rightarrow \{1,2,3, \cdots, 2n\}$

$f(u_i) = i, \; 1 \leq i \leq 2n$

The induced edge labeling is $f^*: E(G) \longrightarrow \{0,1\}$ defined by

$f^*(uv) = 1$, if $gcd(S, D) = 1$,

$f^*(uv) = 0, otherwise$

The edge sets are

$E_1 = \{u_{2i-1} u_{2i+1} / 1 \leq i \leq n-1\}$

$E_2 = \{u_{2i} u_{2i+2} / 1 \leq i \leq n-1\}$

$E_3 = \{u_{2i} u_{2i+1} / 1 \leq i \leq n-1\}$

$E_4 = \{u_{2i+1} u_{2i+2} / 1 \leq i \leq n-1\}$

The induced edge labels are

$$GCD[S, D] = GCD \; [f(u_{2i-1}) + f(u_{2i+1}), |f(u_{2i-1}) - f(u_{2i+1})|]$$

$$= GCD \; [2i - 1 + 2i + 1, |2i - 1 - (2i + 1)|]$$

$$= GCD \; [4i, |-2|]$$

$$= GCD \; [4i, 2], \quad 1 \leq i \leq n-1$$

$$\neq 1$$

$$f^*(u_{2i-1} u_{2i+1}) = 0, \quad 1 \leq i \leq n-1$$

$$GCD\ [S,D] = GCD\ [f(u_{2i}) + f(u_{2i+2}), |f(u_{2i}) - f(u_{2i+2})|]$$

$$= GCD\ [2i + 2i + 2, |2i - (2i + 2)|]$$
$$= GCD\ [4i + 2, |2i - 2i - 2|]$$

$$= GCD\ [4i + 2,2],\quad 1 \le i \le n - 1$$
$$= 0,$$

$$\Rightarrow f^*(u_{2i}u_{2i+2}) = 0,\quad 1 \le i \le n - 1$$

In E_3

$$GCD\ [S,D] = GCD\ [f(u_{2i}) + f(u_{2i+1}), |f(u_{2i}) - f(u_{2i+1})|]$$

$$= GCD\ [2i + 2i + 1, |2i - (2i + 1)|]$$
$$= GCD\ [4i + 1, |2i - 2i - 1|]$$

$$= GCD\ [4i + 1,1],\quad 1 \le i \le n - 1$$
$$= 1$$

$$\Rightarrow f^*(u_{2i}u_{2i+1}) = 1,\quad 1 \le i \le n - 1$$

In E_4

$$GCD\ [S,D] = GCD\ [f(u_{2i+1}) + f(u_{2i+2}), |f(u_{2i+1}) - f(u_{2i+2})|]$$

$$= GCD\ [2i + 1 + 2i + 2, |2i - (2i + 2)|]$$

$$= GCD\ [4i + 3, |2i + 1 - 2i - 2|]$$

$$= GCD\ [4i + 3,1],\quad 1 \le i \le n - 1$$
$$= 1$$

$$\Rightarrow f^*(u_{2i+1}u_{2i+2}) = 1,\quad 1 \le i \le n - 1$$

Thus $|e_{f^*(0)} - e_{f^*(1)}| \le 1$

Hence $O(TL_n)$ admits SD-Prime cordial labeling.

$\therefore O(TL_n)$ is SD prime cordial.

Illustration 2.4

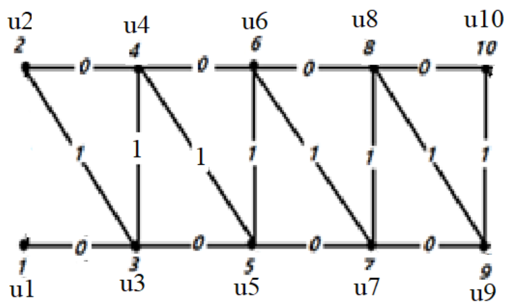

Figure 2 $O(TL_5)$

Theorem 2.5. The graph $D_2(P_n)$ is SD-Prime cordial.

proof: Let $G = D_2(P_n)$

Let

$V(G) = \{u_i/1 \le i \le n\} \cup \{v_i/1 \le i \le n\}$

$E(G) = \{u_iu_{i+1}/1 \le i \le n-1\} \cup \{v_iv_{i+1}/1 \le i \le n-1\} \cup \{u_iv_{n-i}/1 \le i \le n-1\}$
$\cup \{u_{i+1}v_{n-i+1}/1 \le i \le n-1\}$

$|V(G)| = 2n \quad |E(G)| = 4n - 4$

Define a vertex labeling

$f: V(D_2(P_n)) \to \{1,2,3,\cdots,2n\}$ by

$f(u_i) = i,\ 1 \le i \le n$

$f(v_i) = n + i,\ 1 \le i \le n$

The induced edge labeling is $f^*: E(G) \longrightarrow \{0,1\}$ defined by

$f^*(uv) = 1$, if $gcd(S,D) = 1$,

$f^*(uv) = 0, otherwise$

The edge sets are,

$E_1 = \{u_iu_{i+1}/1 \le i \le n - 1\}$

$E_2 = \{v_iv_{i+1}/1 \le i \le n - 1\}$

$E_3 = \{u_iv_{n-i}/1 \le i \le n - 1\}$

$E_4 = \{u_{i+1}v_{n-i+1}/1 \le i \le n - 1\}$

In E_1

$$GCD\ [S,D] = GCD\ [f(u_i) + f(u_{i+1}), |f(u_i) - f(u_{i+1})|]$$

$$= GCD\ [i + (i + 1), |i - (i + 1)|]$$

$$= GCD\ [2i + 1,1],\quad 1 \le i \le n - 1$$
$$= 1$$
$$f^*(u_iu_{i+1}) = 1,\quad 1 \le i \le n - 1$$

In E_2

$$GCD\ [S,D] = GCD\ [f(v_i) + f(v_{i+1}), |f(v_i) - f(v_{i+1})|]$$

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$= GCD\ [(n+i+n+i+1), |n+i-(n+i+1)|]$

$= GCD\ [2n+2i+1, |-1|]$

$= GCD\ [2(n+i))+1, 1], \quad 1 \le i \le n-1$

$= 1$

$f^*(v_iv_{i+1}) = 1, \quad 1 \le i \le n-1$

In E_3

$GCD\ [S,D] = GCD[f(u_i)+f(v_{n-i}), |f(u_i)-f(v_{n-i})|]$

$= GCD[i+n+n-i, |i-(n+n-i)|]$

$= GCD[2n, |i-2n+i|]$

$= GCD[2n, 2(i-n)], \quad 1 \le i \le n-1$

$\neq 1$

$f^*(u_iv_{n-i}) = 0, \quad 1 \le i \le n-1$

In E_4

$GCD\ [S,D] = GCD\ [f(u_{i+1})+f(v_{n-i+1}), |f(u_{i+1})-f(v_{n-i+1})|]$

$= GCD\ [i+1+n+n-i+1, |1+1(n+n-i+1)|]$

$= GCD\ [2n+2, |i+1-2n+i-1|]$

$= GCD\ [2(n+1), 2(i-n)], \quad 1 \le i \le n-1$

$\neq 1$

$f^*(u_{i+1}v_{n-i+1}) = 0$

Thus $|e_{f^*(0)} - e_{f^*(1)}| \le 1$

Hence $D_2(Pn)$ admits SD-Prime cordial labeling.

$\therefore D_2(Pn)$ is SD-Prime cordial

Illustration 2.6

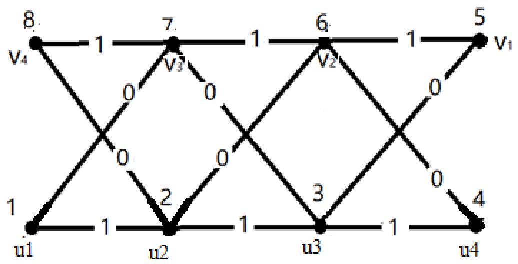

Figure $D_2(Pn)$

Theorem2.7. The graph $D_2(C_n)$, $n \ge 3, n\ \ odd$ is SD Prime cordial.

Proof: Let G be $[D_2(C_n)]$, $n \ge 3, n\ \ odd$

Let $V[D_2(C_n)] = \{u_i, v_i / 1 \le i \le n\}$

and

$E_2[D_2(C_n)] = \{[(u_iu_{i+1}) \cup (v_iv_{i+1})/1 \le i \le n-1] \cup$

$[(u_1u_n) \cup (v_1v_n) \cup (u_1v_2) \cup (v_1u_2)] \cup$

$[(u_iv_{i+1}) \cup (u_iv_{i-1}) \cup (v_iu_{i+1}) \cup (v_iu_{i-1})/2 \le i \le n-1]\}$

$|V(D_2(C_n))| = 2n; \quad |E(D_2(C_n))| = 4n.$

Define a vertex labeling, $f: V(D_2(C_n)) \to \{1,2,3,\cdots,2n\}$ by

$f(u_i) = i, \ 1 \le i \le n$

$f(v_i) = n+i, \ 1 \le i \le n$

The induced edge labeling is $f^*: E(G) \longrightarrow \{0,1\}$ defined by

$f^*(uv) = 1$, if $gcd(S,D) = 1$,

$f^*(uv) = 0, otherwise$

The induced edge labels are,

$GCD\ [S,D] = GCD\ [f(u_i)+f(u_{i+1}), |f(u_i)-f(u_{i+1})|]$

$= GCD\ [i+i+1, |i-(i+1)|]$

$= GCD\ [2i+1, 1], \quad 1 \le i \le n-1$

$= 1$

$f^*(u_iu_{i+1}) = 1, \quad 1 \le i \le n-1$

$GCD\ [S,D] = GCD\ [f(v_i)+f(v_{i+1}), |f(v_i)-f(v_{i+1})|]$

$= GCD\ [n+i+n+i+1, |n+1-(n+i+1)|]$

$= GCD\ [2n+2i+1, |n+i-n-i-1|]$

$= GCD\ [2(n+i)+1, 1], \quad 1 \le i \le n-1$

$= 1$

$f^*(v_iv_{i+1}) = 1, \quad 1 \le i \le n-1$

$GCD\ [S,D] = GCD\ [f(u_i)+f(v_{i+1}), |f(u_i)-f(v_{i+1})|]$

$= GCD\ [i+n+i+1, |i-(n+i+1)|]$

$= GCD\ [2i+n+1, |-n-1|], \quad 2 \le i \le n-1$

$\neq 1$

$f^*(u_i v_{i+1}) = 0, \quad 2 \le i \le n-1$

$GCD\ [S,D] = GCD[f(u_i)+f(v_{i-1}), |f(u_i)-f(v_{i-1})|]$

$= GCD\ [i+n+i-i, |i-(n+i-1)|]$

$= GCD\ [n+2i-1, |i-n-i+1|]$

$= GCD\ [2i+n-1, |n+1|], \quad 1 \le i \le n$
$\neq 1$
$f^*(u_i v_{i-1}) = 0, \quad 1 \le i \le n$

$GCD\ [S,D] = GCD\ [f(v_i)+f(u_{i+1}), |f(v_i)-f(u_{i+1})|]$

$= GCD\ [n+i+i+1, |n+i-(i+1)|]$

$= GCD\ [n+2i+1, |n+i-i-1|]$

$= GCD\ [n+2i+1, |n-1|], \quad 1 \le i \le n$
$\neq 1$
$f^*(v_i u_{i+1}) = 0, \quad 1 \le i \le n$

$GCD\ [S,D] = GCD\ [f(v_i)+f(u_{i-1}), |f(v_i)-f(u_{i-1})|]$

$= GCD\ [n+i+i-1, |n+i-(i-1)|]$

$= GCD\ [n+2i-1, |n+i-i+1|]$

$= GCD\ [n+2i-1, |n+1|], \quad 1 \le i \le n$
$\neq 1$
$f^*(v_i u_{i-1}) = 0, \quad 1 \le i \le n$

Similarly

$f^*(u_1 u_n) = 0$
$f^*(v_1 v_n) = 1$
$f^*(v_1 u_2) = 0$
$f^*(u_1 v_2) = 0$

$|e_{f^*(0)} - e_{f^*(1)}| \le 1$

Hence $D_2(C_n)$ admits SD-Prime cordial labeling.

$\therefore D_2(C_n)$ is SD-Prime cordial

Illustration 2.8

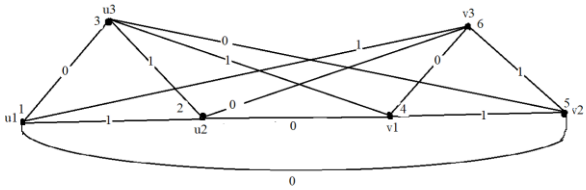

Figure 4 $D_2(C_6)$

Conclusion:

We proved $B_n, O(TL_n), D_2(P_n)$ and $D_2(C_n)$ if $n \ge 3, n\ odd$ are SD-Prime cordial graphs. It is interesting work. Some one may extend for other graphs in future.

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