Finite Element Analysis of Composite Laminate By Using ABDH Matrix(Stiffness Matrix)

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Abstract— Composite laminate have been increasingly used in various field, Predicting and preventing failure of composite becomes impotent aspect now days. This paper is devoted to introduction of ABDH Matrix which is Stiffness Matrix for composite laminate. By using help of stiffness matrix stress and Deformation can be easily find out. ABDH Matrix can be finding out easily by mathematically, but it is very complicated to find for composite laminate with maximum number of ply. So that to find ABDH matrix in minimum time CADEC 10 Software is used. This Stiffness matrix is dependent upon number of layer, thickness, and orientation and material properties.
Keywords- *Composite laminate,ABDH Matrix,CADEC,Finite element Analysis*

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I. INTRODUCTION

Laminated composite material have found extensive application in mechanical,aerospace,marine due to high fatigue life and strength. Fiber and matrix is two main constituent of composites. Mechanical properties of composite material depend upon:

- Properties of Constituent material
- Orientation of each layer
- Thickness of each layer
- Number of ply
- Sequence of Stacking

ABDH Matrix is stiffness matrix of composite laminate dependent upon mechanical properties mentioned above; it is combination of four matrices which help to give information of shear strain, transverse shear strain, bending stiffness.
So here is to check how number of ply impact on the stiffness matrix and its parameter. ABDH matrix also helps to devolved relationship of mechanical load applied to strains and deformation in composite laminates.

II. MATHEMATICAL DERIVATION FOR ABDH MATRIX

Composite laminate is an organized stack of various numbers of piles with random orientation as shown in Fig.1 Here we outline how such laminates are designed and analyzed.

Figure 1: Typical Composite Laminate

While doing analysis on plates of composite laminate axial force and bending moments are acting on it so that we begin by that assuming knowledge of the Force N and moments M applied to a plate at a position x; y, as shown in Fig. 2:

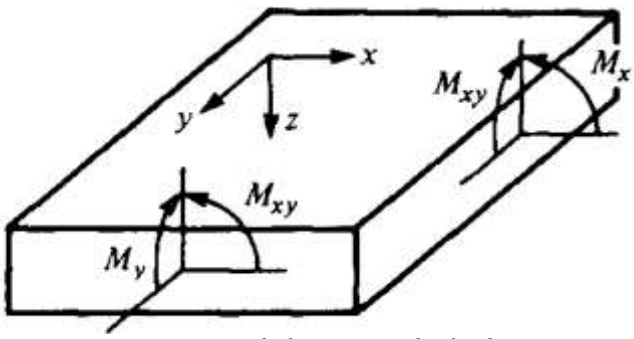

Figure 2: Applied moments in plate bending

$$N=\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} \qquad M=\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} \qquad (1)$$

Co-ordinate x and y are the direction in the plane of the plate And z is assumed positive in downward direction. The reflection in z-direction is noted as w and also assumed positive in downward direction.

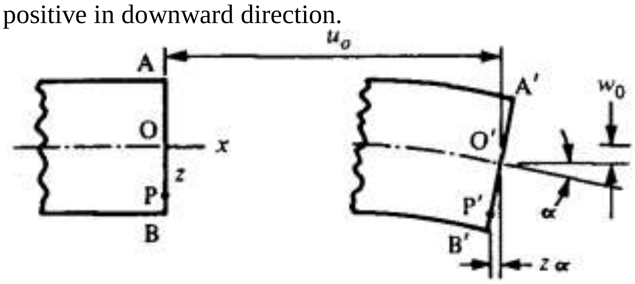

Figure 3: Displacement of mid plane point in the plate

By considering Kirchhoff assumption a line originally straight and perpendicular to the middle surface and remains straight after the deformed (Line A-B in figure). But rotate around mid-plane (Z=0). As shown in Fig.3 the horizontal displacement u and v in the x and y directions due to rotation can be taken to a reasonable approximation from the rotation angle and distance from mid-plane, and this rotational displacement is added to the mid-plane displacement (u_0, v_0) is below.

$$u = u_0 - z\, w_0 x$$
$$v = v_0 - z\, w_0 y \qquad (2)$$

Above equations are of strains at any point in the plate is a function of the displacement, by using matrix notation these can be written as follow:

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} \qquad (3)$$

Where, ε^0_x, ε^0_y, γ^0_{xy} are called the mid-surface strains. They represent the stretching and shear of the plate, and are defined as

$$\varepsilon^0_x(x,y) = \frac{\partial u_0}{\partial x}$$
$$\varepsilon^0_y(x,y) = \frac{\partial v_0}{\partial y} \qquad (4)$$
$$\gamma^0_{xy}(x,y) = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}$$

The curvatures of the plate due to bending k_x and k_y, and due to twisting k_{xy}, are given by

$$k_x(x,y) = -\frac{\partial \phi_x}{\partial x}$$
$$k_y(x,y) = -\frac{\partial \phi_y}{\partial y} \qquad (5)$$
$$k_{xy}(x,y) = -\left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x}\right)$$

Now Stress Relative to each x-y axes are now determined from the strain with the consideration of each ply having their stiffness depends upon their own properties and also their orientation with respect to x-y axes. Therefore Stresses are given below

$$\sigma(z) = [Q][\epsilon^0] + z[Q][k] \qquad (6)$$

Where Q is the transformed stiffness of ply at the position at which stresses are being calculated. Now the stresses are integrated over the thickness of the plate to obtain the resultant forces and moments on a laminate

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \int_{-t/2}^{t/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} dz$$
$$\begin{Bmatrix} V_x \\ V_y \end{Bmatrix} = \int_{-t/2}^{t/2} \begin{Bmatrix} \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} dz \qquad (7)$$
$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-t/2}^{t/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} dz$$

Where N_x, N_y, and N_{xy} are the tensile and shear forces per unit length along the boundary of the plate element with units, V_x, and V_y are the shear forces per unit length of the plate , and M_x, M_y, and M_{xy} are the moments per unit length.

The integrations in equation (7) span over several laminate. Therefore, the integrals can be divided into summations of integrals over each lamina.

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}^k dz$$
$$\begin{Bmatrix} V_x \\ V_y \end{Bmatrix} = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix}^k dz \qquad (8)$$
$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix}^k z\, dz$$

Where k is the lamina number counting from the bottom up, N is the number of laminate in the laminate, and z_k is the coordinate of the top surface of the k_{th} lamina.

From equation (7) and (8) the complete relation is applied force and moment and the resulting mid-plane strain and curvature can be summarized as,

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{Bmatrix} A & B \\ B & D \end{Bmatrix} \begin{Bmatrix} \epsilon^0 \\ k \end{Bmatrix} \qquad (9)$$

Where The A-B-B-D matrix in brackets is the laminate stiffness matrix and its inverse will be the laminate compliance matrix. This can be evaluated as

$$A_{ij} = \sum_{k=1}^{N} (\bar{Q}_{ij})_k (h_k - h_{k-1}) = \sum_{k=1}^{N} (\bar{Q}_{ij})_k t_k$$
$$B_{ij} = \frac{1}{2}\sum_{k=1}^{N} (\bar{Q}_{ij})_k (h_k^2 - h_{k-1}^2) = \sum_{k=1}^{N} (\bar{Q}_{ij})_k t_k \bar{h}_k$$
$$D_{ij} = \frac{1}{3}\sum_{k=1}^{N} (\bar{Q}_{ij})_k (h_k^3 - h_{k-1}^3) = \sum_{k=1}^{N} (\bar{Q}_{ij})_k \left(t_k \bar{h}_k^2 + \frac{t_k^3}{12}\right)$$
$$H_{ij} = \frac{5}{4}\sum_{k=1}^{N} (\bar{Q}^*_{ij})_k \left[t_k - \frac{4}{t^2}\left(t_k \bar{h}_k^2 + \frac{t_k^3}{12}\right)\right] \qquad (10)$$

Summary of ABDH Matrix is given below

[A]=Plate stiffness matrix it direct related to plate strain $(\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0)$ to plane forces (N_x, N_y, N_{xy}).

[B]=Bending extension coupling Matrix relates in plane strain to bending moments curvature to in plane forces.

[D]=Bending stiffness matrix it relates to curvature (k_x, k_y, k_{xy}) to bending moment (M_x, M_y, M_{xy}).

[H]=Transverse shear matrix relate to transverse shear strain $(\gamma_{yz}, \gamma_{xz})$ to transverse shear forces (V_y, V_x).

III PROBLEM STATEMENT FOR COMPOSITE LAMINATE

Find out effect of number of ply over composite laminate considering simply supported square plate A_x=A_y=2000mm, thickness 10mm laminated with AS4D/ 9310 in a [0/ 90] n configuration. Plate is loaded in compression with the edge load N_x= -1N/mm and N_y =N_{xy} =M_x =M_y =M_{xy}=0

Using symmetry to model 1/ 4 of the plate & use of A, B, D and H matrices we try to find out effect of number of ply over composite laminate.

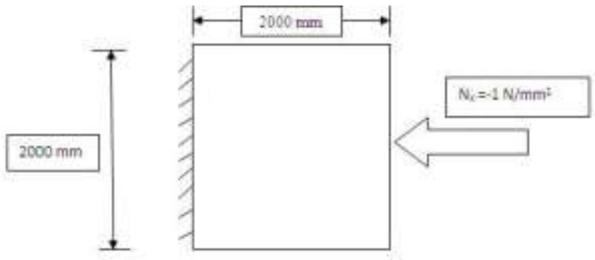

Figure 4: Simply Supported Composite Laminate Plate

Material properties of composite laminate are given in following table:

TABLE I MATERIAL PROPERTIES OF AS4D/9310

Property	AS4D/9310
E_1	133.86 Gpa
E_2=E_3	7.7076 Gpa
G_{12}=G_{13}	4.306 Gpa
G_{23}	2.76 Gpa
V_{12}=V_{13}	0.301
V_{23}	0.396
F_{1t}	1830 Mpa
F_{1c}	1096 Mpa
F_{2t}= F_{3t}	57 Mpa
F_{2c}= F_{3c}	228 Mpa
F_6	71 Mpa

Solution:

A .Procedure for Finite element Analysis

Following procedure was adopted for Finite element Analysis of composite laminate.

1) First step is to calculate ABDH matrix using above input parameter for N=1, 5,10,15,20 mathematically.

ABDH matrix for N=1 is calculated from equations 10 is given below

$$\begin{bmatrix} A & B \\ B & D \end{bmatrix} =$$

TABLE II ABDH MATRIX FOR N=1

7.115e5	2.332e4	0.000	-1.585e6	0.000	0.000
2.332e4	7.115e5	0.000	0.000	1.585e6	0.000
0.000	0.000	4.360e4	0.000	0.000e0	0.000
-1.585e6	0.000	0.000	5.929e6	1.944e5	0.000
0.000	1.585e6	0.000	1.941e5	5.929e6	0.000
0.000	0.000	0.000	0.000	0.000	3.633e5

[H] =

2.966e4	0.000
0. .000	2.966e4

Similarly for different number of ply above matrix is calculated.

2) ABDH matrix using CADEC 10 software :

ABDH matrix is easily calculated by mathematical approach, but it's being difficult as number of ply increases, Hence for ABDH matrix with large number of ply with minimum time and higher accuracy using software CADEC 10. Accuracy of this software is very high which gives error less than 0.5% Material properties are taken as input to CADEC software. By following procedure and taking N=1 ABDH matrix is calculated by using CADEC First all details of the composite laminate is defined into CADEC Software which is shown below

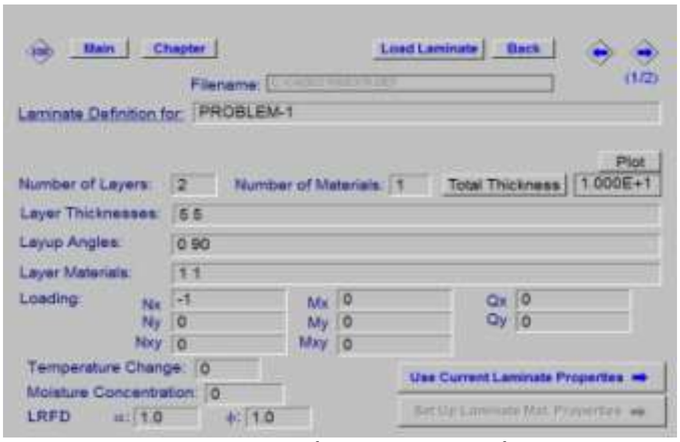

Figure 5: Laminate Definition in CADEC software

After defining all laminate conditions next step is to define Material properties from TABLE into laminate properties which is shown as

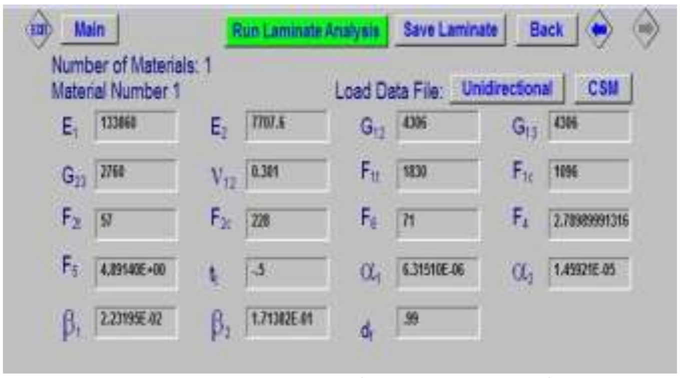

Figure 6: Material properties Definition in CADEC software

After giving all details click on Run laminate analysis tab from same CADEC window, then ABDH Matrix is easily found out using this software which is shown as

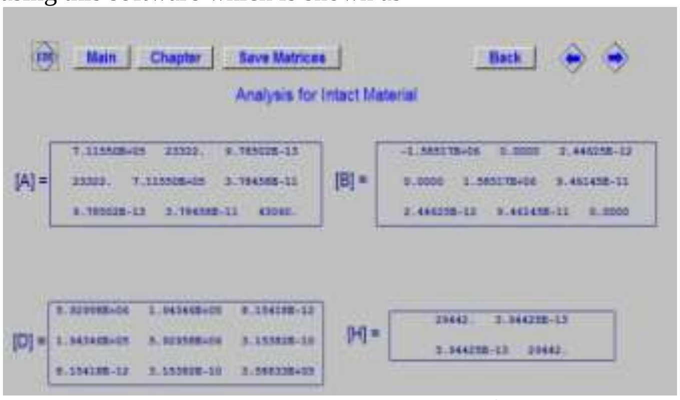

Figure 7: ABDH matrix using CADEC software

ABDH matrix obtained in Figure 7 is similar to table II so it is relevant to use CADEC 10 to find out ABDH matrix for further calculation. Similarly just by changing Laminate definition and Material Properties ABDH matrix is found out with minimum time.

3) Model is created in ANSYS by Using APDL programming, and then meshing is done with element size of 100 mm.

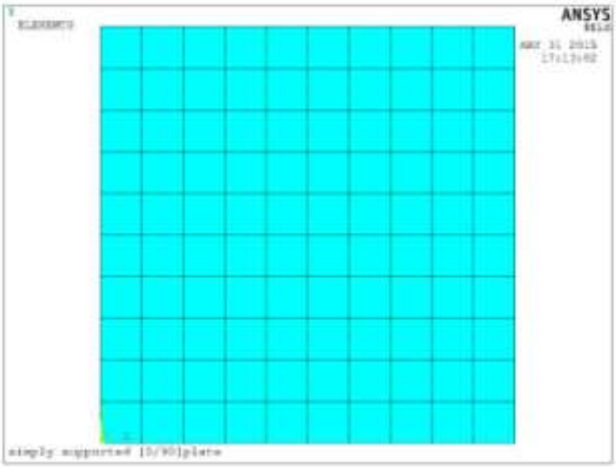

Figure 8: Model of Simply supported [0/90] plate

Then by applying Boundary condition problem is solved by using ANSYS APDL.
Total deformation for plate with N=1 is 0.219144 mm. Deformation plot is shown below

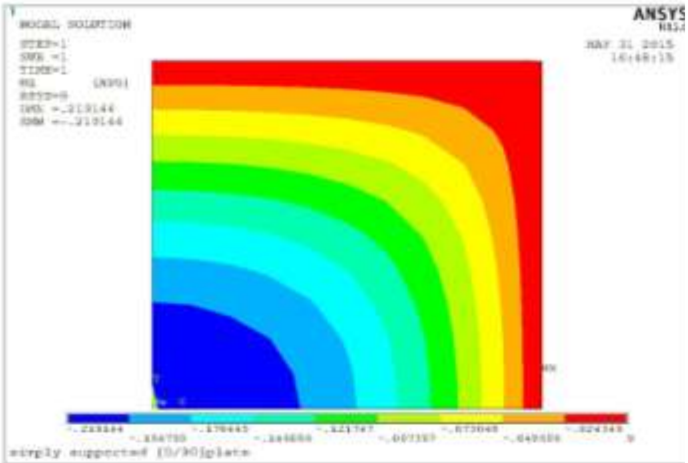

Figure 9: Deformation Plot for N=1

4) Results for different number of ply is calculated and their deformation plot is shown below

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Total deformation for plate with N=5 is 0.21061mm

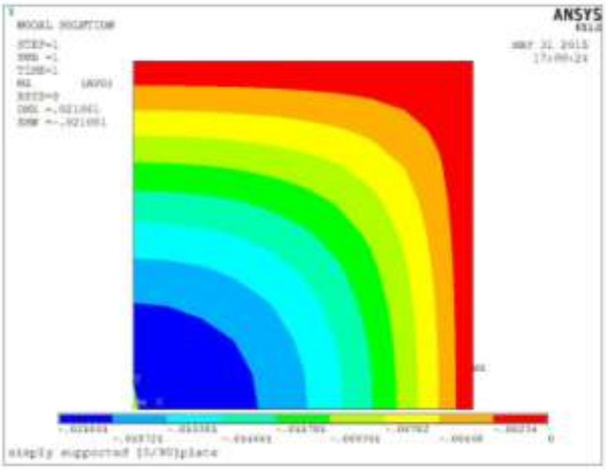

Figure 10: Deformation Plot for N=5

Total deformation for plate with N=10 is 0.10362mm

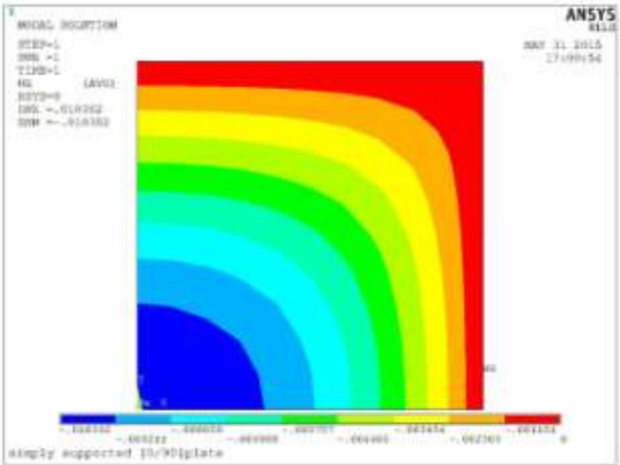

Figure 11: Deformation Plot for N=10

Total deformation for plate with N=15 is 0.006888mm

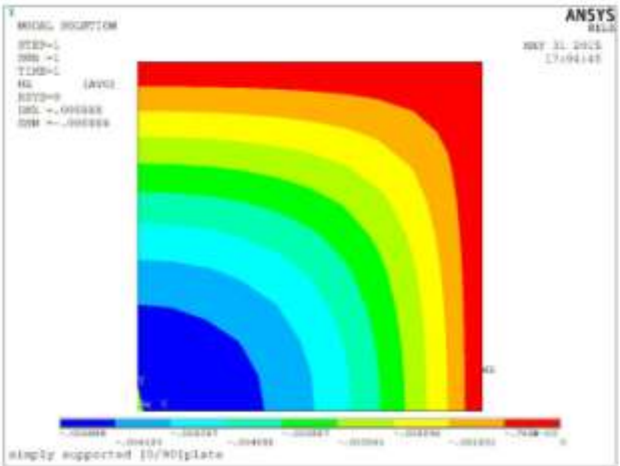

Figure 12: Deformation Plot for N=15

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Total deformation for plate with N=20 is 0.00516mm

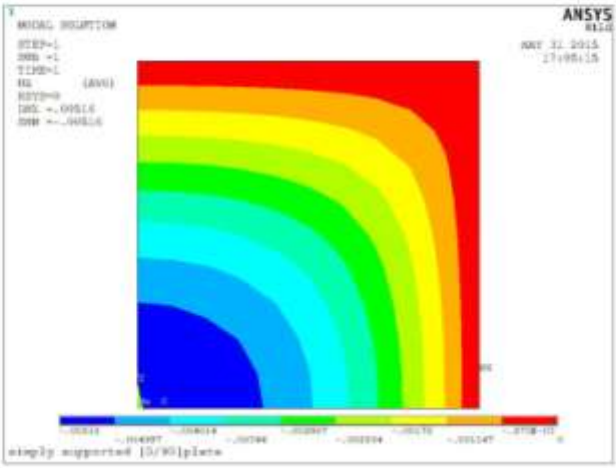

Figure 13: Deformation Plot for N=20

5) Comparison of deformation for different number of ply is mentioned in table shown below

TABLE III COMPARISION OF RESULT

PLATE	DEFORMATION
Plate With N=1	0.21944
Plate With N=5	0.021061
Plate With N=10	0.010362
Plate With N=15	0.00688
Plate With N=20	0.00516

Graph for Deformation for N=1, 5,10,15,20 is shown below

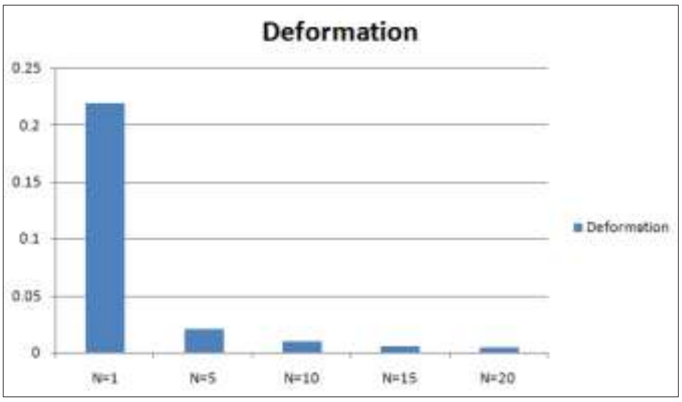

Figure 14: Deformation comparison graph

III. CONCLUSIONS

1) ABDH matrix is Stiffness matrix which gives all information relates to composites laminate.
2) ABDH matrix is function of thickness, stacking sequence, orientation and material properties.
3) CADEC 10 helps to find stiffness matrix in minimum time for large number of ply composites
4) Bending Stiffness decreases by taking maximum number of layers.
5) For maximum strength large number of layers is used with minimum thickness.
6) Deformation decreases as number of layers increases.

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